June 2006 6684 Statistics S2 Mark Scheme

Question Number		Scheme		Marks	
1.	(a)	Saves time / cheaper / easier any one or <u>A census/asking all members</u> takes a long time or is expensive or difficult to carry out	B1	(1)	
	(b)	<u>List, register or database</u> of <u>all</u> club <u>members/golfers</u> or <u>Full membership list</u>	B1	(1)	
	(c)	Club <u>member(s)</u>	B1	(1)	
				rks	
2.	(a)	P(L < -2.6) = $1.4 \times \frac{1}{8} = \frac{7}{40}$ or 0.175 or equivalent	B1	(1)	
	(b)	P (L < -3.0 or L > 3.0) = $2 \times \left(1 \times \frac{1}{8}\right) = \frac{1}{4}$ M1 for 1/8 seen	M1;A1	(2)	
	(c)	P (within 3mm) = $1 - \frac{1}{4} = 0.75$ B(20,0.75) recognises binomial	B1		
		Using B(20,p) Let X represent number of rods within 3mm	M1		
		$P(X \le 9/p = 0.25)$ or $1 - P(X \le 10/p = 0.75)$	M1		
		= 0.9861 awrt 0.9861	A1 Total 7 ma	(4) rks	

Question Number	Scheme	Marks
3. (a)	Let <i>X</i> represent the number of properties sold in a week	
	$\therefore X \sim P_o(7)$ must be in part a	B1
	Sales occur independently/randomly, singly, at a constant rate context needed once	B1 B1 (3)
(b)	P (X = 5) = P(X \le 5) - P(X \le 4) or $\frac{7^5 e^{-7}}{5!}$	M1
	= 0.3007 - 0.1730 = 0.1277 awrt 0.128	A1 (2)
(c)	$P(X > 181) \approx P(Y \ge 181.5)$ where $Y \sim N(168, 168)$ N(168, 168)	B1
	$= P\left(z \ge \frac{181.5 - 168}{\sqrt{168}}\right) \qquad \qquad \begin{array}{c} \pm 0.5 \\ \text{stand with } \mu \text{ and } \sigma\end{array}$	M1 M1
	Give A1 for 1.04 or correct expression = $P(z \ge 1.04)$	A1
	= 1 - 0.8508 attempt correct area 1-p where $p > 0.5$	M1
	= 0.1492 awrt 0.149	A1 (6)
		Total 11 marks

Question Number	Scheme	Marks
4. (a)	Let <i>X</i> represent the number of breakdowns in a week.	
	$X \sim P_0$ (1.25) implied	B1
	P ($X < 3$) = P (0) + P(1) + P(2) or P ($X \le 2$)	M1
	$= e^{-1.25} \left(1 + 1.25 + \frac{(1.25)^2}{2!} \right)$ = 0.868467 awrt 0.868 or 0.8685	A1 A1 (4)
(b)	H ₀ : $\lambda = 1.25$; H ₁ : $\lambda \neq 1.25$ (or H ₀ : $\lambda = 5$; H ₁ : $\lambda \neq 5$) λ or μ	B1 B1
	Let <i>Y</i> represent the number of breakdowns in 4 weeks Under H_0 , <i>Y</i> ~P ₀ (5) may be implied	
	$P(Y \ge 11) = 1 - P(Y \le 10)$ or $P(X \ge 11) = 0.0137$	M1
	One needed for M $P(X \ge 10) = 0.0318$	
	$= 0.0137$ CR $X \ge 11$	A1
$0.0137 < 0.025, 0.0274 < 0.05, 0.9863 > 0.975, 0.9726 > 0.95$ or $11 \ge 11$ any allow %		M1
	$\sqrt{\text{ from H}_1}$ Evidence that the rate of breakdowns has changed /decreased context	B1√ (7)
	From their p	

Question Number	Scheme	Marks
5. (a)	Binomial	B1 (1)
	Let <i>X</i> represent the number of green mugs in a sample	
(b)	$X \sim B$ (10, 0.06) may be implied or seen in part a	B1
	P (X = 3) = ${}^{10}C_3(0.06)^3(0.94)^7$ ${}^{10}C_3(p)^3(1-p)^7$	M1
	= 0.016808 awrt 0.0168	A1 (3)
(c) (i)	Let <i>X</i> represent number of green mugs in a sample of size 125	
	$X \sim P_0(125 \times 0.06 = 7.5)$ may be implied	B1
	$P(10 \le X \le 13) = P(X \le 13) - P(X \le 9)$	M1
	= 0.9784 - 0.7764	
	= 0.2020 awrt 0.202	A1 (3)
(ii)	$P(10 \le X \le 13) \approx P(9.5 \le Y \le 13.5)$ where $Y \sqcup N(7.5, 7.05)$ 7.05	B1
	9.5, 13.5	B1 M1
	$= P\left(\frac{9.5 - 7.5}{\sqrt{7.05}} \le z \le \frac{13.5 - 7.5}{\sqrt{7.05}}\right) \qquad \qquad \pm 0.5 \text{ stand.}$	M1 M1
	both values or both correct expressions	
	= $P(0.75 \le z \le 2.26)$ awrt 0.75 and 2.26	A1
	= 0.2147 awrt 0.214or 0.215	A1 (6) Total 13 marks

Question Number			Marks	
6 (a	$\int_{1}^{4} \frac{l+x}{k} dx = 1$	$\int f(x) = 1$ Area = 1	M1	
	$\therefore \left[\frac{x}{k} + \frac{x^2}{2k}\right]_1^4 = 1$	correct integral/correct expression	A1	
	$k = \frac{21}{2} *$	CSO	A1 (3)	
(b	P(X ≤ x ₀) = $\int_{1}^{x_0} \frac{2}{21} (1+x)$	$\int f(x)$ variable limit or +C	M1	
	$= \left[\frac{2x}{21} + \frac{x^2}{21}\right]_1^{x_0}$	correct integral + limit of 1 May have <i>k</i> in	A1	
	$= \frac{2x_0 + x_0^2 - 3}{21} \text{ or } \frac{(3+x)(x-1)}{21}$		A1	
	$F(x) = \begin{cases} 0, & x < 1\\ \frac{x^2 + 2x - 3}{21} & 1 \le x < 4\\ 1 & x \ge 4 \end{cases}$	middle; ends	B1√; B1 (5)	
(c	$E(X) = \int_{1}^{4} \frac{2x}{21} (1+x) dx$	valid attempt $\int x f(x)$	M1	
	$= \left[\frac{x^2}{21} + \frac{2x^3}{63}\right]_1^4$	x^2 and x^3 correct integration	A1	
	$=\frac{171}{63} = 2\frac{5}{7} = \frac{19}{7} = 2.7142$	awrt 2.71	A1 (3)	

Question Number	Scheme	Marks	
(d)	$F(m) = 0.5 \implies \frac{x^2 + 2x - 3}{21} = \frac{1}{2}$ putting their $F(x) = 0.5$	M1	
	$\therefore 2x^2 + 4x \cdot 27 = 0 \text{or equiv}$ $\therefore x = \frac{-4 \pm \sqrt{16 - 4 \cdot 2(-27)}}{4} \qquad \text{attempt their 3 term quadratic}$	M1	
	4 ∴ $x = -1 \pm 3.8078$ awrt 2.81	A1 (3)	
(e)	Mode = 4	B1 (1)	
(f)	$\frac{\text{Mean} < \text{median} < \text{mode}}{\text{Or}} (\Rightarrow \text{negative skew}) \qquad \text{allow numbers} \\ \frac{\text{Mean} < \text{median}}{\text{Mean}}$	B1 (1)	
	w diagram but line must not cross y axis		
	w diagram out mit must not cross y axis	Total 16 marks	

Question Number	Scheme		Marks	
7. (a)	Let <i>X</i> represent the number of bowls with minor defects.			
	:. $X \sim B;(25, 0.20)$	may be implied	B1; B1	
	P $(X \le 1) = 0.0274$ or P(X=0) = 0.0038	need to see at least one. prob for X≤no For M1	M1A1	
	P (X ≤ 9) = 0.9827; ⇒ P(X ≥ 10) = 0.0173	either	A1	
	$\therefore \operatorname{CR} \text{ is } \left\{ X \le 1 \cup X \ge 10 \right\}$		A1	(6)
b)	Significance level = $0.0274 + 0.0173$			
	= 0.0447 or 4.477%	awrt 0.0447	B1	(1)
c)	$H_0: p = 0.20; H_1: p < 0.20;$		B1 B1	
	Let Y represent number of bowls with minor defects			
	Under H ₀ $Y \sim B$ (20, 0.20)	may be implied	B1	
	P ($Y \le 2$) or P($Y \le 2$) = 0.2061 P($Y \le 1$) = 0.0602	either	M1	
	$P(Y \le 1) = 0.0692$ = 0.2061 CR Y \le 1		A1	
	0.2061 > 0.10 or $0.7939 < 0.9$ or $2 > 1$	their p	M1	
	Insufficient evidence to suggest that the proportion of defective bowls has decreased.		В1√	(7)